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## THE MATHEMATICAL AND SIMULINK MODELS FOR DETERMINING ATTITUDE OF AIRCRAFT USING MEMS SENSORS

## МАТЕМАТИЧЕСКАЯ и SIMULINK МОДЕЛИ ОПРЕДЕЛЕНИЯ ПРОСТРАНСТВЕННОГО ПОЛОЖЕНИЯ САМОЛЕТА С ИСПОЛЬЗОВАНИЕМ MEMS-ДАТЧИКОВ

# МЕМЅ-ДАТЧИКТЕРДІ ҚОЛДАНА ОТЫРЫП, ҰШАҚТЫҢ КЕҢІСТІКТІК ЖАҒДАЙЫН АНЫҚТАУДЫҢ МАТЕМАТИКАЛЫҚ ЖӘНЕ SIMULINK МОДЕЛЬДЕРІ

**Abstract.** This paper analyses the features, types and principles of construction of miniature gyroscopes and accelerometers, as one of the main types of sensors of inertial measuring systems. An analysis is also being made of the possibilities and prospects for integrating magnetometers and GPS modules into inertial computing systems, which significantly improvers the performance of the measuring system. The physical principles and mathematical models of MEMS accelerometers, gyroscopes, magnetometers and their integrated modules are presented, which make it possible to build miniature magnetic heading and attitude meters for aircraft with high accuracy. A typical MEMS model with an accelerometer and a gyroscope is considered, a mathematical model and a MatLab/Simulink model of such a MEMS sensor system are given, which makes it possible to build a real model of system for measuring the attitude and course of aircraft.

**Keywords:** MEMS sensors, accelerometers, gyroscopes, magnetometers, inertial computing systems, mathematical model, attitude, Simulink model.

Аннотация. В данной работе анализируются особенности, виды и принципы построения миниатюрных гироскопов и акселерометров, как одного из основных видов датчиков инерциальных измерительных систем. Также проводится анализ возможностей и перспектив интеграции магнитометров и GPS модулей в инерциальные вычислительные системы, благодаря чего значительно улучшаются показатели измерительной системы. Приводятся физические принципы и математические модели MEMS акселерометров, гироскопов, магнитометров и их интегрированных модулей, позволяющих построить миниатюрных измерителей магнитного курса и пространственного положения летательных аппаратов с высокой точностью. Рассматривается типовая модель MEMS с акселерометром и гироскопом, дается математическая модель и Matlab/Simulink модель такой MEMS системы датчиков, что позволяет построить реальную модель системы измерения пространственного положения и курса летательных аппаратов.

Ключевые слова: MEMS-датчики, акселерометры, гироскопы, магнитометр, инерциальные вычислительны системы, математическая модель, пространственное положение, Simulink модель.

Аңдатпа. Бұл жұмыста инерциялық өлшеу жүйелерінің датчиктерінің негізгі түрлерінің бірі ретінде миниатюралық гироскоптар мен акселерометрлердің ерекшеліктері, турлері және құрылыс принциптері талданады. Сондай-ақ магнитометрлер мен GPS модульдерін инерциялык есептеу жуйелеріне біріктіру мумкіндіктері мен перспективаларына талдау жүргізілуде, бұл өлшеу жүйесінің өнімділігін айтарлықтай жақсартады. MEMS акселерометрлерінің, гироскоптарының, магнитометрлерінің және олардың біріктірілген модульдерінің физикалық принциптері мен математикалық үлгілері ұсынылған, бұл ұшақтар үшін жоғары дәлдікпен миниатюралық магниттік айдар мен қатынас өлшегіштерін құруға мүмкіндік береді. Акселерометрі мен гироскопы бар типтік MEMS моделі қарастырылды, мұндай MEMS сенсорлық жүйесінің математикалық моделі және Matlab/Simulink моделі келтірілген, бұл MEMS датчиктерінің қатынасы мен барысын өлшеуге арналған жүйенің нақты моделін құруға мүмкіндік береді. ұшақ.

Түйін сөздер: MEMS сенсорлары, акселерометрлер, гироскоптар, магнитометр, инерциялық есептеу жүйелері, математикалық модель, қатынас, Simulink моделі.

**Introduction**. An inertial measurement unit (IMU) is a group of sensors consisting of an accelerometer measuring acceleration and a gyroscope measuring angular velocity. Frequently, a magnetometer is also included to measure the Earth's magnetic field. Each of these three sensors produces a 3-axis measurement, and these three measurements constitute a 9-axis measurement.

IMU and GPS sensor fusion to determine orientation and position. Use inertial sensor fusion algorithms to estimate orientation and position over time. The algorithms are optimized for different sensor configurations, output requirements, and motion constraints. The addition of a magnetometer and filtering algorithms to determine orientation information results in a device.

Vibrations of the accelerometer will accumulate error and cause drift in any velocity or position calculations, a term called vibration rectification. Drift in the offset of an accelerometer measurement due to temperature changes or other sources of error will accumulate over time.

Gyroscopes are an essential component of any instrument rig used for attitude, heading, turning, and navigation. Technological inventions created gyroscopes that work using a variety of theories. Each type of gyro is best suited for particular situations based on the type of information needed and the effect of drift.

MEMS Gyro-Micro-Electro-Mechanical Systems gyros work based on the Coriolis force. As the MEMS chip is subjected to angular acceleration, Coriolis forces impart a displacement of a vibrating plate. This force is translated into electrical signals. At the high end, these are found in many aircraft AHRS, and at the low end in many consumer electronics.

IMUs can measure a variety of factors, including speed, direction, acceleration, specific force, angular rate, and (in the presence of a magnetometer), magnetic fields surrounding the device.

IMUs combine input from several different sensor types in order to accurately output movement. An IMU is a specific type of sensor that measures angular rate, force and sometimes magnetic field. IMUs are composed of a 3-axis accelerometer and a 3-axis gyroscope, which would be considered a 6-axis IMU. They can also include an additional 3-axis magnetometer, which would be considered a 9-axis IMU [1].

Each tool in an IMU is used to capture different data types:

- Accelerometer: measures velocity and acceleration

- Gyroscope: measures rotation and rotational rate

- Magnetometer: establishes cardinal direction (directional heading), measurement of the magnetic field surrounding the system.

Accelerometer is a sensor that measures the specific force (the body mass normalizes the force). It provides the acceleration across the x, y, and z axes in its local frame. MEMS Accelerometers measure linear acceleration (specified in mV/g) along one or several axis. MEMS accelerometers are used wherever there is a need to measure linear motion, either movement, shock or vibration but without a fixed reference. They measure the linear acceleration of whatever they are attached to.

There are two types of piezoelectric accelerometers (vibration sensors). The first type is a "high impedance" charge output accelerometer. In this type of accelerometer the piezoelectric crystal produces an electrical charge which is connected directly to the measurement instruments. The charge output requires special accommodations and instrumentation most commonly found in research facilities. This type of accelerometer is also used in high temperature applications (>120C) where low impedance models can not be used. The second type of accelerometer is a low impedance output accelerometer. A low impedance accelerometer has a charge accelerometer as its front end but has a tiny built-in micro-circuit and FET transistor that converts that charge into a low impedance voltage that can easily interface with standard instrumentation.

Gyroscope is a sensor that measures angular velocity around the x, y, and z axes, in its local frame. Generally, integrating the measurements results in the angles themselves. A gyroscope measures angular velocity (specified in mV/deg/s).

Magnetometer is a sensor that measures the Earth's magnetic field and provides the heading (the compass is one such device) and is included in the IMU.

A gyroscope is an inertial sensor that measure an object's angular rate with respect to an inertial reference frame. There are many different types of gyroscopes available on the market, which range over various levels of performance and include mechanical gyroscopes, fiber-optic gyroscopes (FOGs), ring laser gyroscopes (RLGs), and quartz/MEMS gyroscopes. Quartz and MEMS gyroscopes are typically used in the consumer grade, industrial grade, and tactical grade markets, while fiber-optic gyroscopes span all four of the performance categories. Ring laser gyroscopes typically consist of in-run bias stabilities ranging anywhere from 1 °/hour down to less than 0.001 °/hour, encompassing the tactical and navigation grades. Mechanical gyroscopes make up the highest performing gyroscopes available on the market and can reach in-run bias stabilities of less than 0.0001 °/hour.

**MEMS Accelerometers.** An accelerometer is the primary sensor responsible for measuring inertial acceleration, or the change in velocity over time, and can be found in a variety of different types, including mechanical accelerometers, quartz accelerometers, and MEMS accelerometers. A MEMS accelerometer is essentially a mass suspended by a spring, as illustrated in **Figure** 1a. The mass is known as the proof mass and the direction that the mass is allowed to move is known as the sensitivity axis.



When an accelerometer is subjected to a linear acceleration along the sensitivity axis, the acceleration causes the proof mass to shift to one side, with the amount of deflection proportional to the acceleration.

Now consider that the accelerometer is rotated such that the sensitivity axis is aligned with the gravity vector, as shown in Figure 1b. In this case, gravity acts on the proof mass causing it to deflect downward. Due to this, the accelerometer measures both the linear acceleration due to motion as well as the pseudo-acceleration caused by gravity. The acceleration caused by gravity is referred to as a pseudo-acceleration as it does not actually result in a change in velocity or position [2].

In the coordinate frame shown in Figure 1b, the pseudo-acceleration caused by gravity is measured as a -1 g, as gravity has the same effect on the accelerometer as an acceleration due to motion in the negative z-axis. It is also important to note that during free fall, the springs in the accelerometer do not deflect, and consequently the sensor reports an acceleration of zero, though the actual acceleration is non-zero.

**MEMS Gyroscopes.** Typically, MEMS sensing structures range from 1 micrometer to 100 micrometers. MEMS gyroscopes use a vibrating element for rate measurement. The underlying principle is, any vibrating body has a tendency to continue vibrating in its plane of vibration. As a consequence, if the orientation of the platform to which a vibrating body is attached is changed, the vibrating body will exert a force on the platform. This force can be measured and can be used to find out the output.

A gyroscope is an inertial sensor that measure an object's angular rate with respect to an inertial reference frame. MEMS gyroscopes measures the angular rate by applying the theory of the Coriolis effect, which refers to the force of inertia that acts on objects in motion in relation to a rotating frame. To better understand, consider a mass suspended on springs, as illustrated in Figure 2a. This mass has a driving force on the x-axis causing it to oscillate rapidly in the x-axis. While in motion an angular velocity,  $\omega$ , is applied about the z-axis. This results in the mass experiencing a force in the y-axis as a result of the Coriolis force, and the resultant displacement is measured by a capacitive sensing structure.



(a) Single Mass

(b) Tuning Fork Configuration

Figure 2. Simple Gyroscope Model

The classic gyroscope consists of a spinning wheel or disc which works based on the principle of conservation of angular momentum. The rotation of the spinning axis remains unaffected due to the conservation of the momentum. As the technology evolved, other types of gyroscopes were developed which could provide more accurate and consistent output. Over the period, as potential applications for gyroscopes were identified, need to develop low cost and compact gyroscopes were felt. This lead to the development of MEMS gyroscopes which are compact in size and give an adequately good performance. These gyros exploit the effects of the Coriolis forces which are experienced when a vibrating mass is subjected to a rate of rotation about an axis in the plane of vibration. There are two basic configurations which are being exploited; a tuning fork configuration and a vibrating cylinder configuration (Figure 2b).

To achieve the highest accuracy requires accurate modelling in the system computer of the temperature dependent errors. Several manufacturers use quartz tuning fork rate gyros for deriving the aircraft attitude in their solid-state standby display instruments – a good testimony to their performance and attributes of small size, low power consumption, very high reliability and competitive cost.

It is important to note that the accelerometer results provide accurate orientation angles as long as gravity is the only force acting on the sensor. However, when moving and rotating the sensor, we are applying forces to it, which causes the measurements to fluctuate. The net result is that accelerometer data tends to be very noisy, with brief but significant perturbations. If these can be averaged out, the accelerometer provides accurate results over timescales longer than the perturbations.

Computing orientation from the gyroscope sensor is different, since the gyroscope measures angular velocity (the rate of change in orientation angle), not angular orientation itself. To compute the orientation, we must first initialize the sensor position with a known value (possibly from the accelerometer), then measure the angular velocity ( $\omega$ ) around the X, Y and Z axes at measured intervals ( $\Delta t$ ). Then  $\omega \times \Delta t$  = change in angle. The new orientation angle will be the original angle plus this change. The problem with this approach is that we are integrating – adding up many small computed intervals – to find orientation. Repeatedly adding up increments of  $\omega \times \Delta t$  will result in small systematic errors becoming magnified over time. This is the cause of gyroscopic drift, and over long timescales the gyroscope data will become increasingly inaccurate.

Advantages of MEMS gyroscopes over FOG/RLG:

1. Extremely space efficient. Available in the form of chips, so can be fitted on electronic circuits.

2. Adequate performance. As the technology is evolving, the performance accuracy of MEMS gyroscopes is also improving.

3. No moving components unlike DTG (dynamically-tuned)/RLG and hence, completely maintenance free.

4. Available at a fraction of the cost of FOG or RLG.

**MEMS Magnetometers.** A magnetometer is a type of sensor that measures the strength and direction of a magnetic field. While there are many different types of magnetometers, most MEMS magnetometers rely on magnetoresistance to measure the surrounding magnetic field. Magnetoresistive magnetometers are made up of permalloys that change resistance due to changes in magnetic fields [3]. Typically, MEMS magnetometers are used to measure a local magnetic field which consists of a combination of Earth's magnetic field as well as any magnetic fields created by nearby objects.





(a) Standard dipole magnet (b) Earth's magnetic field Figure 3. Dipole Approximation of Earth's Magnetic Field

As illustrated in Figure 3, the Earth's magnetic field is a self-sustaining magnetic field that resembles a magnetic dipole with the geomagnetic poles slightly offset from the geographic North and South poles. This magnetic field is characterized by a strength and direction, which varies across the Earth and can shift over time. The direction of the Earth's magnetic field contains a horizontal component as well as a vertical component and is often described using the magnetic inclination and declination angles. Magnetic inclination describes the angle between the Earth's magnetic field lines and a horizontal plane. At the Earth's magnetic field is horizontal at the equator and has an inclination angle of 90 °, whereas the Earth's magnetic field is horizontal at the geographic North Pole of the Earth is not in the same location as True North or the geographic North Pole of the Earth and is characterized as the angle between these two locations, relative to the point of measurement [4,5].

Attitude estimation. In a stationary accelerometer setup, the magnitude of the acceleration is constant. Since the magnitude is constant, the accelerometer can be used to determine the attitude of the system. Using only an accelerometer, and the constant magnitude of gravity, it is possible to determine two of the three angles needed for a complete estimation of the attitude, as shown by equation 2 and 3.

$$|\vec{a}| = \sqrt{\vec{a_x^2} + \vec{a_y^2} + \vec{a_z^2}} = 1g$$
(1)

roll = 
$$\beta = \theta_{\rm x} = {\rm sign}\left(\frac{\overline{a_{\rm y}}}{\overline{a_{\rm z}}}\right) * {\rm arctan}\left(\sqrt{\frac{|\overline{a_{\rm y}}|^2}{|\overline{a_{\rm x}}|^2 + |\overline{a_{\rm z}}|^2}}\right)$$
 (2)

pitch = 
$$\gamma = \theta_y = \text{sign}\left(\frac{\overrightarrow{a_x}}{\overrightarrow{a_z}}\right) * \arctan\left(\sqrt{\frac{|\overrightarrow{a_x}|^2}{|\overrightarrow{a_y}|^2 + |\overrightarrow{a_z}|^2}}\right)$$
 (3)

In contrast to an accelerometer, a gyroscope is a MEMs device which measures the angular velocity of a system. It is typical for a gyroscope to be a 3-axis gyroscope, which gives the ability to detect angular change across the three axis – roll, pitch, yaw. In a single gyroscope setup, it is possible to determine the angle of the system by integrating the angular change over the duration of the rotation.

$$roll = \beta = \theta_x = \int_0^t \omega_x dt$$
(4)

pitch = 
$$\gamma = \theta_y = s \int_0^t \omega_y dt$$
 (5)

These basic methods are not very useful in attitude estimation due to their individual drawbacks when used in a practical environment. In the case of using the accelerometer only to determine the attitude, the attitude calculation is susceptible to movements and vibrations and the noisy nature of the accelerometer. In addition, calculating the angle using an accelerometer will not give an indication of the yaw.

In Figure 4, at time t = 0, the axes OX, OY, OZ are as shown in dotted lines. At time t = $\Delta t$ , the angular rotations of the vector components are  $p\Delta t$ ,  $q\Delta t$ ,  $r\Delta t$  respectively. The corresponding changes in the vectors are thus  $-U_q\Delta t$ ,  $U_r\Delta t$ ,  $V_p\Delta t$ ,  $-V_r\Delta t$ ,  $-W_p\Delta t$ ,  $W_q\Delta t$ . The rate of change of these vectors, that is, the centripetal acceleration components are thus: -Uq, Ur, Vp, -V r, -Wp, Wq.

The changes in the velocity components due to the change in magnitude of the velocity vector are  $\Delta U$ ,  $\Delta V$ ,  $\Delta W$  respectively. The rates of change of these velocity components are  $\Delta U/t$ ,  $\Delta V/t$ ,  $\Delta W/t$ , which in the limit becomes dU/dt, dV/dt, dW/dt.

The linear acceleration components are thus:

Acceleration along 
$$OX = U - V r + Wq$$
 (6)

Acceleration along  $OY = \dot{V} + Ur - Wp$  (7)

Acceleration along 
$$OZ = \dot{W} - Uq + Vp$$
 (8)

The angular acceleration components about OX, OY and OZ are  $\dot{p}$ ,  $\dot{q}$ , and  $\dot{r}$  respectively. The dot over the symbols denotes d/dt and is Newton's notation for a derivative. Thus  $\dot{U} = dU/dt$ and  $\dot{p} = dp/dt$ , etc. The motion of the aircraft can then be derived by solving the differential equations of motion obtained by applying Newton's second law of motion in considering the forces and moments acting along and about the OX, OY and OZ axes respectively [6].

Namely, the rate of change of momentum is equal to the resultant force acting on the body, i.e.

### force = mass $\times$ acceleration

In the case of angular motion, this becomes:

moment (or torque) = (moment of inertia)  $\times$  (angular acceleration)



Figure. 4 Vector change in velocity components due to angular rotation

Figure 5. Euler angles

The relationship between the angular rates of roll, pitch and yaw (Figure 5) p, q, r (by gyros) and the Euler angles,  $\Psi$ ,  $\theta$ ,  $\Phi$  and the Euler angle rates  $\dot{\Psi}$ ,  $\dot{\theta}$ ,  $\dot{\Phi}$  are derived as follows:

Consider Euler bank angle rate,  $\dot{\Phi}$ 

$(OX = \Phi)$	
Component of $\dot{\Phi}$ along $OY = 0$	
OZ = 0	
Consider Euler pitch angle rate, $\dot{\theta}$	
OX = 0	
Component of $\dot{\theta}$ along $OY = \dot{\theta} \cos \Phi$	
$OZ = -\dot{\Theta}\sin\Phi$	
Consider Euler yaw angle rate, $\dot{\Psi}$	
$(OX = -\dot{\Psi}\sin\theta)$	
Component of $\Psi$ along $OY = \Psi$ cos $\theta$ sin $\Phi$	
$0.07 = \dot{\Psi}_{cos}\theta_{cos}\Phi$	
Hence,	
$\mathrm{p}=\dot{\Phi}$ - $\dot{\Psi}\sin heta$	(9)
$q = \dot{\theta} \cos \Phi + \dot{\Psi} \cos \theta \sin \Phi$	(10)
$r = \dot{\Psi} \cos\theta \cos\Phi - \dot{\theta} \sin\Phi$	(11)

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The Euler angles,  $\Phi$ ,  $\theta$ ,  $\Psi$  are defined according to Figure 4 and 5 and the equations relating the body angular rates p, q, r to the Euler angle rates are derived and comprise equations (9), (10), (11). Equations for  $\Phi$ ,  $\theta$ ,  $\Psi$  can be derived from these equations by suitable algebraic manipulation and are set out below:

$$\dot{\Phi} = p + q \sin\Phi \tan\theta + r \cos\Phi \tan\theta \tag{12}$$

$$\dot{\theta} = q \cos \Phi - r \sin \Phi \tag{13}$$

$$\Psi = q \sin \Phi \sec \theta + r \cos \Phi \sec \theta \tag{14}$$

These equations can be expressed more compactly in matrix form

$$\begin{aligned} \dot{\Phi} \\ \dot{\theta} \\ \dot{\Psi} \end{aligned} = \begin{bmatrix} 1 & \sin\Phi \tan\theta & \cos\Phi \tan\theta \\ 0 & \cos\Phi & -\sin\Phi \\ 0 & \sin\Phi \sec\theta & \cos\Phi \sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$
(15)

The mathematical and Simulink model for MEMS gyro and accelerometer. The behavior of MEMS accelerometer is a typical enforced mass-dapmer-spring system. We can represent by the equation (Figure 6):

$$\mathbf{F} = \mathbf{m}\mathbf{x}'' + \mathbf{k}\mathbf{x} + \mathbf{c}\mathbf{x}' \tag{16}$$



Figure 6. The typical MEMS sensor model

This physical model shows a typical MEMS vibratory gyroscope designed to measure the angular velocity of the body about the z axis of the ground reference frame. The main principle of MEMS gyros is the transfer of energy between two modes of vibration through the Coriolis acceleration.

In principle we can use either accelerometer or gyroscope to estimate the tilt angle of the object. Gyroscope measures angular velocity, thus we must integrate the output with respect to time to obtain the tilt angle. For a typical digital output MEMS sensor the output data is in integer format, and for analog output MEMS sensor it is a voltage. Thus we must know how to interface with the digital MEMS sensor or converts the voltage outputs of the analog MEMS accelerometer and gyroscope to angle and angular velocity. Of course, there are already many online resources that describe MEMS the usage of these sensors. To obtain the tilt angle estimate from the gyroscope, the angular velocity needs to be integrated, this is shown below.

$$\theta_{\rm G}(t) = \int_0^t \omega_{\rm G}(t) d\tau = \theta_{\rm b}(t) + \int_0^t e_{\rm G}(\tau) d\tau \tag{17}$$

 $\int_0^t e_G(\tau) d\tau$  this term will grow as time progress. System of two equation:

$$mx'' + k_{x}x + c_{x}x' = F_{0}sin(\omega_{d}t)$$
(18)  
$$my'' + k_{y}y + c_{y}y' = -2m\omega x'$$
(19)

$$y'' + k_y y + c_y y' = -2m\omega x'$$
 (19)

The first equation can be solved independently. You can then substitute its solution into the second equation, which can be solved for the sense-mode response y. The resulting amplitudes of motion X and Y are given below;  $Y/\omega$  is the gyroscope sensitivity: Where:

$$X = \frac{F_0}{m} \omega_x \frac{1}{\sqrt{(1 - r_x^2)^2 + (2\zeta_x r_x)^2}} \quad \omega_{x=\sqrt{\frac{k_x}{m}}, \qquad \zeta_x = \frac{c_x}{2m\omega_x}, \qquad r_x = \frac{\omega_d}{\omega_x}$$
(20)

$$\frac{Y}{\omega} = \left(\frac{2m\omega_d X}{k_y}\right) \frac{1}{\sqrt{(1-r_y^2)^2 + (2\zeta_y r_y)^2}} \qquad \qquad \omega_{y=\sqrt{\frac{k_y}{m}}, \qquad \zeta_y = \frac{c_y}{2m\omega_y}, \qquad r_y = \frac{\omega_d}{\omega_y}$$
(21)

The accelerometer and gyroscope output are shown below. For accelerometer output:

$$\theta_{A=}\theta_{b} + e_{A} \tag{22}$$

 $\theta_b$  is actual titl angle and  $e_A$  sensor errors. For gyroscope output:

$$\omega_{\rm G} = \frac{\rm d}{\rm dt} \theta_{\rm b} + e_{\rm G} \tag{23}$$

 $\theta_b$  is actual titl angle and  $e_G$  sensor errors. MEMS output will be:

The sensitivity is proportional to the oscillating mass, which puts some restrictions on the level of miniaturization that can be achieved. To achieve maximum sensitivity, resonance in both modes is desirable; that is,  $\omega_d = \omega_x = \omega_y$ . The expression for the gyroscope sensitivity then becomes

$$\frac{Y}{\omega} = \frac{2m\omega_d F_0 Q_x Q_y}{k_x k_y} \qquad Q_x = \frac{1}{2\zeta_x} \qquad Q_y = \frac{1}{2\zeta_y}$$
(24)

 $Q_x$  and  $Q_y$  are the quality factors of the drive and sense modes, respectively. High-quality factors are desirable to improve sensitivity.

In Figure 7, considering aforementioned drawn mathematical equations with MEMS input and output for attitude determining of aircraft, the following Simulink models shown in diagram for sensors [7].

#### Conclusion

This article describes how we can use a magnetometer, accelerometer, and a gyro to estimate an airctaft's orientation. MEMS sensors have been reviewed and major specifications described in this paper. Accelerometer and gyro mathematical formulas were shown. The goal of Simulink model is to show how these sensors contribute to the solution, and to explain a few moments to watch out for along the way. It is covered what orientation is and how we can determine attitude using an accelerometer, gyroscope and a magnetometer. The conducted reseches show that using modern MEMS sensors and integrating them into a mesuring system, it is possible to development miniature attitude and position determination system with improved technical performance.





Figure 7. Matlab/Simulink model for MEMS gyro and accelerometer

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